HOMEWORK SET 12: ZEEMAN EFFECT II
Due Wednesday, February 26, 2025

PROBLEMS FROM TZDII[1]

9.22) a) Use Eq. (9.37) (with the Bohr values $L = 2\hbar$ and $r = 4a_B$) to show that the fine structure separation $\Delta E_{FS} = 2\mu_B B$ of the two 2p levels of hydrogen can be written as

$$B = \frac{\mu_0}{4\pi}\frac{eL}{m_e r^3} \quad (9.37)$$

$$\Delta E_{FS} = \frac{m_e\left(ke^2\right)^4}{32\hbar^4 c^2} \quad (9.38)$$

[*Hint*: Since $\mu_0\varepsilon_0 = 1/c^2$ and $k = 1/4\pi\varepsilon_0$, you can replace μ_0 in 9.37]

b) Show that you can rewrite (9.38) as $\Delta E_{FS} = \frac{\alpha^2 E_R}{16}$ (9.39)

where α is the dimensionless *fine-structure constant*, $\alpha = \frac{ke^2}{\hbar c}$ (9.40)

c) Show that $\alpha \approx 1/137$, which, together with (9.39), shows that the fine structure is indeed a small effect.

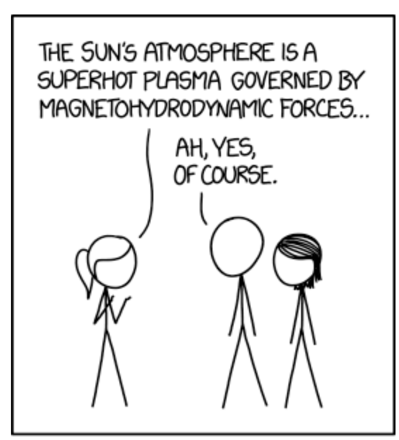
9.21 Extra Credit) The fine structure of an atomic spectrum results from the magnetic field "seen" by an orbiting electron. In this question you will make a smeiclassical estimate of the B field seen by a 2p electron in hydrogen. The B field at the center of a circular current loop, i, of radius r is known to be $B = \mu_0 i/2r$.

a) Treating the electron an proton as classical particles in circular orbits (each as seen by the other), show that the B field seen by the elctron is given by (9.37 above) where L is the electron's orbital angular momentum (L = mvr for a circular orbit). Remember that the current produced by the orbiting proton is $i = ev/2\pi r$, where v is the speed of the proton as seen by the elctron (or vise versa). (HINT: APPLY THE BIOT-SAVART LAW TO THE PROTON AT THE CENTER OF A CURRENT LOOP CREATED BY THE ELECTRON AND INTEGRATE AROUND THE LOOP. THE CURRENT IS THE CHARGE/TIME = E/(PERIOD OF ROTATION).)

$$dB = \frac{\mu_0}{4\pi}\left|\frac{id\vec{\ell}\times\hat{r}}{r^2}\right| = \frac{\mu_0}{4\pi}\frac{id\vec{\ell}}{r_p^2} \quad \left(\hat{r}\perp \text{to } d\vec{\ell}\right)$$

b) For a rough estimate, you can give L and r their values for the n = 2 orbit of the Bohr model, $L = 2\hbar$ and $r = 4a_B$. Show that this gives $B \approx 0.39$T and hence that the separation, $2\mu_B B$, of the two 2p levels is about 4.5×10^{-5} eV.

It should be clear that this semiclassical calculation is only a rough estimate. Comment on why this is so (read the entire problem in TZDII and explain why this is clear).



WHENEVER I HEAR THE WORD "MAGNETOHYDRODYNAMIC" MY BRAIN JUST REPLACES IT WITH "MAGIC."

[1] Taylor, Zafiratos, & Dubson, *Modern Physics for Scientists and Engineers*, 2nd Editon, Pearson, Prentice Hall, 2004